

Programme	BSc Mathematics Honours			
Course Code	MAT4CJ203			
Course Title	REAL ANALYSIS I			
Type of Course	Major			
Semester	IV			
Academic Level	200 – 299			
Course Details	Credit	Lecture/Tutorial per week	Practicum per week	Total Hours
	4	3	2	75
Pre-requisites	1. Mathematical Logic and necessary exposure to set theory. 2. Basic Calculus			
Course Summary	After introducing the basic notions in set theory, the course develops into the construction of the Real number system. Thereafter Real functions are introduced and the notions of limit and continuity are developed.			

Course Outcomes (CO):

CO	CO Statement	Cognitive Level*	Knowledge Category#	Evaluation Tools used
CO1	Demonstrate Proficiency in Set Theory Fundamentals and Real Number Properties	An	C	Internal Exam/ Assignment/Seminar/ Viva/Report/ End Sem Exam
CO2	Apply the completeness property of \mathbb{R} , and solve problems involving intervals and applications of the supremum property.	U	C	Internal Exam/ Assignment/Seminar/ Viva/Report/ End Sem Exam
CO3	Analyse sequences and their limits, apply limit theorems, and demonstrate an understanding of concepts such as monotone sequences, sub-sequences, and the Cauchy Criterion, as well as their applications in solving problems related to sequences and limits.	An	C	Internal Exam/ Assignment/Seminar/ Viva/Report/ End Sem Exam
* - Remember (R), Understand (U), Apply (Ap), Analyse (An), Evaluate (E), Create (C) # - Factual Knowledge(F) Conceptual Knowledge (C) Procedural Knowledge (P) Metacognitive Knowledge (M)				

Detailed Syllabus:

Textbook		Introduction to Real Analysis, 4/e, Robert G Bartle, Donald R Sherbert John Wiley & Sons (2011)		
Module	Unit	Content	Hrs (45+30)	External Marks (70)
I	Introduction to Set theory		8	Min.15
	1	Section 1.1 - Sets and functions (for review only)		
	2	Section 1.2 - Mathematical Induction (Proofs of results included in practicum part).		
	3	Section 1.3 – Finite and Infinite sets.		
	4	Section 1.3 – Countable and Uncountable sets.		
II	The Real numbers		13	Min.15
	5	Section 2.1 – The algebraic properties of \mathbb{R} .		
	6	Section 2.1 – The order properties of \mathbb{R} .		
	7	Section 2.2 – Absolute value and the Real Line.		
	8	Section 2.3 – Completeness property of \mathbb{R} (Proofs included in Practicum).		
	9	Section 2.4 – Applications of the Supremum property - 2.4.3 to 2.4.6 and 2.4.8 to 2.4.9 (All other discussions included in Practicum).		
	10	Section 2.5 – Intervals – 2.5.2 to 2.5.4 (All other discussions included in Practicum).		
III	Sequences and Limits		12	Min.15
	11	Section 3.1 – Sequences and their limits.		
	12	Section 3.1 – Problems to find limits of sequence.		
	13	Section 3.2 – Limit theorems.		
	14	Section 3.2 – Problems using Limit theorems.		
	15	Section 3.3 – Monotone sequences – Monotone Convergence Theorem.		
	16	Section 3.3 – Applications of Monotone Convergence Theorem – Euler’s number introduction only.		
IV	Sequences and Limits (continued)		12	Min.10
	17	Section 3.4 – Sub sequences and the Bolzano Weierstrass theorem (Second proof of Theorem 3.4.8 is omitted for external exam and limits superior and inferior are included in practicum).		
	18	Section 3.4 – Problems using Divergence criteria.		
	19	Section 3.5 – The Cauchy Criterion (Examples 3.5.9, 3.5.11 and Corollary 3.5.10 are included in Practicum).		
	20	Section 4.1- Limits of functions (Proofs included in Practicum).		
	21	Section 4.2: Limit theorems of functions (Proofs included in Practicum).		

	22	Section 4.3: Some extensions of limit concepts (Proofs included in Practicum).		
V	Practicum: The goal is for the students to learn the following topics in 15 practicum sessions of two hours each via self-study and group activities. The lecturer may assist by running group discussions, supervising class seminars and referring library books for self-study and note preparation.		30	-
	1	Section 1.2 - for detailed discussions including proofs		
	2	Section 2.3 – re do it with all the proofs		
	3	Section 2.4 – Worked out examples for applying the ideas of supremum and infimum and the existence of square root of 2		
	4	Section 2.5 – Characterization theorem for intervals and representations of real numbers		
	5	Section 3.4 – discussions of limit inferior and limit superior with examples		
	6	Section 3.5 – Estimation of errors in contractive sequences with examples		
	7	Section 3.6 – Properly divergent Sequences		
	8	Section 3.7 – Introduction to Infinite Series – conditions for convergence – Harmonic Series		
	9	Section 3.7 – Comparison Tests with examples		
	10	Section 4.1 – Formulate a precise definition of limit and illustrate with examples		
	11	Section 4.1 – Sequential Criterion for Limits for convergence and divergence with examples		
	12	Section 4.2 – Limit theorems for functions in parallel to that of sequences.		
	13	Section 4.3 – One sided and infinite limits.		
	14	Section 11.1 – Open sets, their properties and characterization.		
	15	Section 11.1 - Closed sets, their properties and characterization.		
References				
<ol style="list-style-type: none"> 1. Tom.M. Apostol, Calculus I, Wiley & Sons. 2. Tom.M. Apostol, Mathematical Analysis, 2/e, Addison-Wesley. 3. Richard R Goldberg, Methods of Real Analysis, 2/e, Wiley 4. Raymond L Wilder, Introduction to the Foundations of Mathematics,2/e, John WileySons 				
Optional Programming References for Practicum:				
<ol style="list-style-type: none"> (1) SageMath Calculus Tutorial https://www.sagemath.org/calctut/limits.html (2) SageMath 2D plotting https://doc.sagemath.org/html/en/reference/plotting/sage/plot/plot.html# 				

***70 external marks are distributed over the first four modules subjected to a minimum of 15 marks from each module.**

Mapping of COs with PSOs and POs:

	PSO1	PSO2	PSO3	PSO4	PO1	PO2	PO3	PO4	PO5	PO6	PO7
CO 1	2	1	3	2	3	0	3	0	3	0	0
CO 2	1	3	2	2	3	0	3	0	3	0	0
CO 3	3	2	3	3	3	0	3	0	3	0	0

Correlation Levels:

Level	Correlation
-	Nil
1	Slightly / Low
2	Moderate / Medium
3	Substantial / High

Assessment Rubrics:

- Assignment/ Report
- Seminar
- Internal Exam
- Viva
- Final Exam (70%)

Mapping of COs to Assessment Rubrics:

	Internal Exam	Assignment/ Report	Seminar	Viva	End Semester Examinations
CO 1	✓	✓	✓	✓	✓
CO 2	✓	✓	✓	✓	✓
CO 3	✓	✓	✓	✓	✓

Programme	BSc Mathematics Honours			
Course Code	MAT4CJ204			
Course Title	BASIC LINEAR ALGEBRA			
Type of Course	Major			
Semester	IV			
Academic Level	200 – 299			
Course Details	Credit	Lecture/Tutorial per week	Practicum per week	Total Hours
	4	4	--	60
Pre-requisites	1. Familiarity with system of equations and their solutions 2. Knowledge about matrices and matrix operations.			
Course Summary	This course is a quick review of linear algebra, intended for students who have already taken a previous course in linear algebra or have some experience with vectors and matrices. It begins with the concepts of vector spaces, subspaces, bases and dimension. Linear transformations are introduced as ‘natural maps’ between vector spaces. The course opens up the classical finite dimensional inner product theory for the canonical reduction of a matrix as a special case of a self-adjoint operator.			

Course Outcomes:

CO	CO Statement	Cognitive Level*	Knowledge Category#	Evaluation Tools used
CO1	Understand and apply concepts related to vector spaces and subspaces, including determining whether a set forms a subspace and finding the span of a set	U	C	Internal Exam/Assignment/Seminar/Viva/ End Sem Exam
CO2	Demonstrate proficiency in analysing null spaces, column spaces, and linear transformations, including understanding the kernel and range of a linear transformation and contrasting the properties of null space and column space.	An	P	Internal Exam/Assignment/Seminar/Viva/ End Sem Exam
CO3	Evaluate and apply concepts related to bases, dimensionality, and rank of vector spaces, including understanding bases for null space and column space, determining dimensions of subspaces, and applying the rank theorem to systems of equations.	E	C	Internal Exam/Assignment/Seminar/Viva/ End Sem Exam
* - Remember (R), Understand (U), Apply (Ap), Analyse (An), Evaluate (E), Create (C) # - Factual Knowledge(F) Conceptual Knowledge (C) Procedural Knowledge (P) Metacognitive Knowledge (M)				

Detailed Syllabus:

Text Book	Linear Algebra and its Applications, Third Edition, David .C. Lay, Pearson Publications				
Module	Unit	Content	Hrs (48+ 12)	External Marks (70)	
I	Module I			14	Min 15
	1	Section 4.1: Vector Spaces and Subspaces Vector Spaces and Subspaces, Subspaces, A Subspace Spanned by a Set.			
	2	Section 4.2: Null Spaces, Column Spaces, and Linear Transformations. The Null Space of a Matrix, An Explicit Description of Nul A.			
	3	Section 4.2: Null Spaces, Column Spaces, and Linear Transformations. The Column Space of a Matrix, The Contrast Between Nul A and Col A.			
	4	Section 4.2: Null Spaces, Column Spaces, and Linear Transformations. Kernel and Range of a Linear Transformation.			
	5	Section 4.3: Linearly Independent Sets; Bases. Linearly Independent Sets; Bases, The Spanning Set Theorem.			
	6	Section 4.3: Linearly Independent Sets; Bases. Bases for Nul A and Col A, Two Views of a Basis.			
II	Module II			12	Min 15
	7	Section 4.4: Coordinate Systems. Coordinate Systems, A Graphical Interpretation of Coordinates, Coordinates in \mathbb{R}^n .			
	8	Section 4.4: Coordinate Systems. The Coordinate Mapping.			
	9	Section 4.5: The Dimension of a Vector Space. The Dimension of a Vector Space.			
	10	Section 4.5: The Dimension of a Vector Space. Subspaces of a Finite-Dimensional Space, The Dimensions of Nul A and Col A.			
	11	Section 4.6: Rank Rank, The Row Space.			
	12	Section 4.6: Rank The Rank Theorem, Applications to Systems of Equations (Topics up to and including Example 5).			
III	Module III			12	Min 15
	13	Section 6.1: Inner Product, Length and Orthogonality The Inner Product, The Length of a Vector, Distance in \mathbb{R}^n .			
	14	Section 6.1: Inner Product, Length and Orthogonality Orthogonal Vectors, Orthogonal Complements, Angles in \mathbb{R}^2 and \mathbb{R}^3 .			
	15	Section 6.2: Orthogonal Sets			

		Orthogonal Sets, An Orthogonal Projection (Topics up to and including Example 4).		
	16	Section 6.2: Orthogonal Sets Orthonormal Sets.		
	17	Section 6.4: The Gram-Schmidt Process The Gram -Schmidt Process, Orthonormal Bases.		
	18	Section 6.4: The Gram -Schmidt Process QR Factorization of Matrices.		
IV	Module IV			
	19	Section 7.1: Diagonalization of Symmetric Matrices Diagonalization of Symmetric Matrices.	10	Min 15
	20	Section 7.1: Diagonalization of Symmetric Matrices The Spectral Theorem. Spectral Decomposition.		
	21	Section 7.2: Quadratic Forms Quadratic Forms (Topics up to and including Example 3), Classifying Quadratic Forms.		
	22	Section 7.4: The Singular Value Decomposition The Singular Value Decomposition, The Singular Values of an $m \times n$ Matrix , The Singular Value Decomposition (Topics up to and including Example 4 only).		
V	OPEN ENDED		12	
	<p>Linear Algebra Lab Sessions</p> <p>Book: Mike Cohen, Practical Linear Algebra for Data Science, O'Reilly, 2019, ISBN 978-1-098-12061-0.</p> <p>Jupyter: https://github.com/mikexcohen/LinAlg4DataScience</p> <p>Choose lab demos and exercises for 12 hours as per lecturer's discretion.</p> <p>For Module I & II, Ch 2, 3, 5, 6 of book for Lab. For Module III, Ch 2 and Ch 9 of book for Lab. For Module IV, Ch 14 of book for Lab. Python and Jupyter review in Ch 16 of book.</p>			
References				
<ol style="list-style-type: none"> 1. Elementary Linear Algebra: Application Version, 11/e, Howard Anton & Chris Rorres Wiley 2. Algebra Done Right, 3/e, Sheldon Axler, Springer Nature, 2015. 3. Introduction to Linear Algebra, 6/e, Gilbert Strang, Wellesley-Cambridge Press. 4. Basic Linear Algebra, 2/e, T. S. Blyth and E.F. Robertson, Springer, 2002. 5. Linear Algebra, 2/e, Hoffman K and Kunze R, Prentice Hall of India, 1991. 6. Bretscher, Otto. <i>Linear algebra with applications</i>. Vol. 52. Eaglewood Cliffs, NJ: Prentice Hall, 1997. 7. Blyth, Thomas Scott, and Edmund F. Robertson. <i>Basic linear algebra</i>. Springer Science & Business Media, 2013. 				

***70 external marks are distributed over the first four modules subjected to a minimum of 15 marks from each module.**

Mapping of COs with PSOs and POs :

	PSO1	PSO2	PSO3	PSO4	PO1	PO2	PO3	PO4	PO5	PO6	PO7
CO 1	2	1	2	2	3	0	3	0	3	0	0
CO 2	1	3	2	2	3	0	3	0	3	0	0
CO 3	3	2	3	3	3	0	3	0	3	0	0

Correlation Levels:

Level	Correlation
-	Nil
1	Slightly / Low
2	Moderate / Medium
3	Substantial / High

Assessment Rubrics:

- Assignment/ Seminar
- Internal Exam
- Viva
- Final Exam (70%)

Mapping of COs to Assessment Rubrics:

	Internal Exam	Assignment	Seminar	Viva	End Semester Examinations
CO 1	✓	✓	✓	✓	✓
CO 2	✓	✓	✓	✓	✓
CO 3	✓	✓	✓	✓	✓

Programme	BSc Mathematics Honours			
Course Code	MAT4CJ205			
Course Title	FUNDAMENTALS OF PYTHON AND SAGEMATH			
Type of Course	Major			
Semester	IV			
Academic Level	200-299			
Course Details	Credit	Lecture/Tutorial per week	Practical per week	Total Hours
	4	3	2	75
Pre-requisites	1) Basic knowledge to start a desktop/laptop computer 2) A basic course in calculus with an understanding of differential and integral calculus (higher secondary level and one or two semester courses from Bsc) 3) A basic course in linear algebra ((higher secondary level))			
Course Summary	In the first part of the course, it intends to give a quick introduction to writing python programs using various popular interfaces. How to handle data and save and read them files is introduced next along with the concepts of repeating the tasks using conditionals and loops. The problems connected with matrices and arrays is solved using the python module numpy. The python module SymPy is used to do various mathematical problems related with symbolic computations. A brief introduction of python module pandas is given, which is used to do data analysis. Using the Python programming structure, an introduction to the advance mathematics software sagemath is given in the second part of the course. Various practical problems making use of concepts from the calculus and linear algebra are to be solved using the sagemath software so that the students will come to know some of the applications of mathematics in real life.			

Course Outcomes (CO):

CO	CO Statement	Cognitive Level*	Knowledge Category #	Evaluation Tools used
CO1	Develop proficiency in fundamental to advanced Python programming concepts, including variables, data types, control structures, functions, modules, file handling, and matrix operations.	C	C	Internal Exam/Quiz/End Sem
CO2	Demonstrate competence in data visualization techniques using Matplotlib, encompassing plotting mathematical functions, 2D and 3D graphics, and animated plots.	Ap	C	Internal Exam /Assignment/ End Sem
CO3	Develop proficiency in symbolic computation with SymPy, data manipulation with Pandas, and algebraic computations with SageMath, enabling them to solve diverse mathematical problems numerically and analytically.	C	C	Internal Exam /viva/ Seminar/End Sem
* - Remember (R), Understand (U), Apply (Ap), Analyse (An), Evaluate (E), Create (C) # - Factual Knowledge(F) Conceptual Knowledge (C) Procedural Knowledge (P) Metacognitive Knowledge (M)				

Detailed Syllabus:

Textbook	<ol style="list-style-type: none"> 1. Ajith Kumar B.P., Python for Education, https://scischool.in/python/pythonForEducation.pdf 2. Gregory V. Bard, Sage for Undergraduates (online version) http://www.people.vcu.edu/~clarson/bard-sage-for-undergraduates-2014.pdf 3. Tuan A. Le and Hieu D. Nguyen, SageMath Advice For Calculus, https://users.rowan.edu/~nguyen/sage/SageMathAdviceforCalculus.pdf 		
Module	Unit	Content	Hrs (45+ 30)
I	Introductory Python and Arrays (Text 1: Chapter 2, Chapter 3)		
	1	Section 2.1: Getting started with Python Section 2.2: Variables and Data Types, Keywords, Section 2.3: Operators and their Precedence.	12
	2	Section 2.4: Python Strings Section 2.5: Python Lists Section 2.6: Mutable and Immutable Types. Section 2.7: Input from the Keyboard Section 2.8: Python Syntax, Colon & Indentation	
	3	Section 2.9: Controlling the Progame Flow Section 2.10: Iteration: for loops Section 2.11: Conditional Execution: if, elif and else Section 2.12: Modify loops: break and continue..	
	4	Section 2.15: Functions Section 2.17: Python Modules and Packages. Section 2.18: File Input/Output Section 2.19: Formatted Printing. Section 2.21: Matrices in pure Python.	
	5	All topics up to Section 3.1, Section: 3.1: NumPy Arrays	
	6	Section: 3.2: Vectorizing Functions.	
II	Data Visualization (Text 1: Chapter 4)		

	7	Section: 4.1: The Matplotlib Module	10
	8	Section: 4.2: Plotting mathematical functions Section: 4.3: Plotting Error Bars, Section: 4.4: Simple 2D animation.	
	9	Section: 4.5: Famous Curves Section: 4.6: 2D plot using colors.	
	10	Section: 4.7: 3D Plots.	
III	Introduction to SymPy and Pandas (Text 1: Chapter 5 and Chapter 6)		10
	11	All topics up to Section 5.1, Section 5.1: SymPy, Symbolic Computation in Python.	
	12	Section 5.2: SymPy, Derivative and Integral	
	13	Section 5.3: SymPy, Operation on sets	
	14	Section 6.1: Series	
	15	Section 6.2: Data Frame	
	16	Section 6.3: Practical Examples	
IV	Sagemath – An Introduction (Text 2: Chapter 1, For units 17,18,19)		13
	17	Getting and installing sagemath in Windows, Ubuntu OS Using sagemath using cocalc (online) Section 1.1: Using Sage as a Calculator Section 1.2: Using Sage with Common Functions Section 1.3 : Using Sage for Trigonometry	
	18	Section 1.5: Matrices and Sage, Part One 1.5.1: A First Taste of Matrices 1.5.3: Doing the RREF in Sage	
	19	Section 1.5: Using Sage to Manipulate Polynomials	
	(Text 3: Chapter 2, 3, 5, For units 20,21,22)		
	20	Section 2.1: Plotting Graphs	
	21	Section 3.1: The Derivative	
		Section 3.2: Higher-Order Derivatives	

22	Section 5.1: Antiderivatives (Indefinite Integral), Section 5.2: Riemann Sums and the Definite Integral All topics up to 5.2.1, 5.2.1: Riemann Sum Using Left Endpoints	
Practical		30
Online References for Practical		
<ol style="list-style-type: none"> 1. Python official website and documentation, https://www.python.org/ 2. Spyder official website and documentation, https://www.spyder-ide.org/ 3. Getting Started: Python and IDLE, MIT Courseware, https://web.mit.edu/6.s189/www/handouts/GettingStarted.html 4. Jupyter Notebook, https://jupyter.org/ 5. Google Colaboratory (colab), https://colab.google/ 6. Pydroid 3 IDE for Android (https://play.google.com/store/apps/details?id=ru.iiec.pydroid3&hl=en_US&pli=1) with Pydroid 3 repository plugin (https://play.google.com/store/apps/details?id=ru.iiec.pydroid3.quickinstallrepo&gl=US). 		
<p>Practical problems in basic Python</p> <ol style="list-style-type: none"> 1) Write a programme to work as a basic Income Tax Calculator 2) Write a program that takes the length of an edge (an integer) as input and prints the cube's surface area as output. 3) Write a loop that counts the number of space characters in a string. Recall that the space character is represented as ' '. 4) Write a while loop that computes the factorial of a given integer N. 		

- 5) Write a program that computes square roots.
- 6) Write a programme for data Encryption based on Caesar shift.
- 7) Develop a program that computes the Flesch Index for a text file.
- 8) Using a List to Find the Median of a Set of Numbers
- 9) Finding the Mode of a List of Values.

Numerical methods using python (Text1: Chapter 7)(7.1 - 7.10, 7.12)

- 1) Evaluate a Taylor series numerically.
- 2) Interpolate a function using
 - a) Newton's forward interpolation
 - b) Newton's backward interpolation
 - c) Lagrange's Interpolation
 - d) Newton's General Interpolation
- 3) Find integral of function using
 - a) Trapezoidal rule
 - b) Simpson's 1/3-rule
- 4) Find derivative of function numerically.
- 5) Solve first order differential equations numerically.
 - a) Euler method
 - b) Fourth order Runge-Kutta method
- 6) Solve algebraic equations numerically.
 - a) The Bisection method
 - b) Regula Falsi Method

Practical problems using numpy, matplotlib, pandas and sympy

- 1) Various vector operations. such as dot product, cross product and divergent using numpy module.
- 2) Various matrix operations such as determinant, inverse and transpose using numpy module.
- 3) Solve system of linear equations using numpy module.
- 4) Plot various 2-D, 3-D curves using matplotlib module.

	<p>5) Plot various 3-D surfaces using matplotlib module.</p> <p>6) Find maxima and minima of a function using SymPy module.</p> <p>7) Necessary data analysis of a given data using pandas module.</p> <p>Practical problems in Sage</p> <p>1) Solve a system of linear equations (Text 2)</p> <p>2) Constrained Optimization by Lagrange Multipliers (Text 2, 4.18.2)</p> <p>3) Traffic Flow (Text 3)</p> <p>4) Minimum Cost (Text 3)</p> <p>5) Packaging (Minimum Surface Area) (Text 3)</p> <p>6) Maximize Revenue (Text 3)</p> <p>7) Area Between Curves (Text 3)</p> <p>8) Average Value and mean value theorem (Text 3, 6.2)</p> <p>9) Newton's Method to find approximate roots (Text 3)</p>
<p>References:</p> <p>1 Amit Saha, Doing Math with Python, No Starch Press, 2015.</p> <p>2 Vernon L. Ceder, The Quick Python Book, Second Edition, Manning.</p> <p>3 Python tutorial online, https://www.geeksforgeeks.org/python-programming-language/</p> <p>4 2D plotting, https://doc.sagemath.org/html/en/reference/plotting/sage/plot/plot.html</p> <p>5 3D Graphics, https://doc.sagemath.org/html/en/reference/plot3d/index.html</p> <p>6 Linear Algebra, https://doc.sagemath.org/html/en/tutorial/tour_linalg.html</p> <p>7 John Harris, Karen Kohl, and John Perry, Peering into Advanced Mathematics through Sage-colored Glasses</p> <p>8 Paul Zimmermann, Alexandre Casamayou, Computational Mathematics with SageMath, https://www.sagemath.org/sagebook/english.html</p> <p>Kenneth A Lambert, Fundamentals of Python First Programs, Edn 2, Cengage</p>	

***70 external marks are distributed over the first four modules subjected to a minimum of 15 marks from each module.**

Mapping of COs with PSOs and POs :

	PSO1	PSO2	PSO3	PSO4	PO1	PO2	PO3	PO4	PO5	PO6	PO7
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CO 2	2	2	3	1	3	2	3	3	1	1	2
CO 3	2	2	3	1	3	2	3	3	1	1	2

Correlation Levels:

Level	Correlation
-	Nil
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Assessment Rubrics:

- Assignment/ Seminar
- Internal Exam
- Viva
- Quiz
- Practical Based Assessment
- Final Exam (70%)

Mapping of COs to Assessment Rubrics:

	Internal Exam	Assignment	Seminar	Quiz	Viva	Practical based assessment	End Semester Examinations
CO 1	√			√		√	√
CO 2	√	√				√	√
CO 3	√		√		√	√	√